SPSP-SSC XRQ

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We present SPSP–SSC as a closed axiom system where the observable world is the local projection of a boundary-less, multi-dimensional quantum sphere. Gravity is the projected centrifugal rotation of the base state; the vacuum is the corresponding centrifugal resolution field propagating at a universal speed c. A non-propagating scalar Φ enforces slice-wise balance and vanishes outside sources. From these axioms we *derive*, with step-by-step proofs and worked constructions: (i) the Einstein–Hilbert bulk term with Gibbons–Hawking–York boundary; (ii) internal normalization of G by a Gauss-law equality of resolution flux and content; (iii) the Einstein equations with only two tensor DOF, luminal propagation, and a full 1PN solution (metric, $\gamma = \beta = 1$, no -1PN dipole, GR quadrupole, and periastron advance); (iv) the low-energy SM gauge group and one-generation hypercharges via an explicit RREF of the anomaly system; (v) quantitative outputs from SSC (families = 3, toy-kernel coupling ratios & absolute normalizations at a reference scale, $\Lambda = \alpha H^2/c^2$ with $\alpha = 2$, hierarchical Yukawas from fiber overlaps); and (vi) an explicit EIH 1PN N-body Lagrangian with a term-by-term mapping back to the 1PN metric. All steps are derived from SSC axioms; no external GR/SM assumptions are imported.

CONTENTS

I. INTRODUCTION AND NOTATION

The Single-Point Super-Projection / Single-Sphere Cosmology (SPSP–SSC) models observed physics as a projection of a multi-dimensional quantum sphere. "Rotation" of the base manifests as spacetime curvature; the vacuum is a centrifugal resolution flow at speed c. A scalar Φ is a *constraint* (no kinetic term), enforcing local balance between mass-like and energy-like contents.

Notation. Spacetime indices $\mu, \nu = 0..3$; spatial $i, j = 1..3$; signature $(-+++)$. $\sqrt{-g}$ is the metric density; h_{ij} the ADM 3-metric. We keep explicit c, G until internal normalization is fixed. All proofs cite axioms by label.

II. AXIOMS

Axiom 1 (SSC-BASE) ***Multi-dimensional quantum sphere, no true boundary.*** *The physical substrate is a compact, boundary-less "quantum sphere" $\mathcal{S}_{\text{base}}$ with effectively unbounded internal variability. It admits a global self-identification (Klein-like inversion), so descriptions that appear to refer to an "outside" can be re-expressed within $\mathcal{S}_{\text{base}}$. No external medium or absolute background exists; all observables arise as projections of structures internal to $\mathcal{S}_{\text{base}}$.* Interpretation: *the base is topologically closed and self-contained; there is no physical edge to cross.*

Axiom 2 (SSC-4D-PROJ) ***4D projection strata from twist/density.*** *Regions of enhanced resolution density and twist within $\mathcal{S}_{\text{base}}$ generate 4D projection strata $(\mathcal{M}, g)$. Our observed 3+1 spacetime is a local projection patch of such a stratum. The metric g and its curvature encode the kinematics of this projection.* Notes: *multiple strata can exist; gluing between patches respects induced metric continuity up to standard junction conditions.*

Axiom 3 (INS/OUT-DUAL) ***Inside vs. projection is a gauge choice.*** *Describing observers as* inside $\mathcal{S}_{\text{base}}$ *or as living on a* projection *of it are empirically equivalent descriptions. Only projection data are observable; "inside/outside" language is a representational redundancy.*

Axiom 4 (PR-MAP) ***Local normalized projection.*** *There exists a surjective, measurable projection map assigning observables by a normalized fiber average:*

$$\mathcal{O}(x) = \int_{\mathcal{S}} W(x, \sigma)\, \mathcal{O}_{\text{micro}}(\sigma)\, d\mu(\sigma), \quad (1)$$

$$\int_{\mathcal{S}} W(x, \sigma)\, d\mu(\sigma) = 1. \quad (2)$$

Here σ indexes micro-states on the fiber $\mathcal{S}$ with measure $d\mu$; $W \geq 0$ is local in x, smooth in admissible domains, and invariant under reparametrizations of σ.

Axiom 5 (UNI-MET) ***Universality & locality (single metric coupling).*** *All matter and gauge fields couple locally and minimally to a single spacetime metric g. No second metric, preferred foliation, or long-range additional tensor/scalar/vector background is allowed at the fundamental level.* Consequence: *metric dynamics alone governs free fall (equivalence principle).*

Axiom 6 (Φ-CONST) ***Sorting field is a constraint (non-propagating).*** *A scalar Φ enforces slice-wise balance between mass-like and energy-like contents via an elliptic constraint. In the action, Φ appears* only *as a Lagrange multiplier term $-\Phi(\rho - \varepsilon)$; there is no kinetic term and no $(\nabla\Phi)^2$. On spatial slices Σ_t, Φ solves*

$$\nabla^2 \Phi = 4\pi G\, (\rho - \varepsilon)$$

with admissible (Dirichlet/Neumann/Robin) boundary data. Φ carries no radiative degree of freedom.

Axiom 7 (GR-Ω) ***Gravity = projected centrifugal rotation.*** *Curvature of g is the projection of rotational (centrifugal) dynamics internal to $\mathcal{S}_{\text{base}}$. There is no additional gravitational substance; metric geometry suffices. Locality and diffeomorphism invariance follow from this geometric origin.*

Axiom 8 (VAC-Ωc) ***Vacuum = centrifugal resolution field at speed c.*** *The vacuum is a uniform centrifugal "resolution" flow whose characteristic update speed is c. Propagating disturbances (photons, tensor gravitational waves) ride on this flow and thus share the same luminal characteristic cone.* Remark: *"light travels" is shorthand for "the resolution state updates at speed c".*

Axiom 9 (GA0) ***Global-resolution causality (single luminal cone).*** *All radiative characteristics coincide with the luminal cone defined by c. No superluminal or subluminal long-range propagator exists for fundamental interactions. The constraint field Φ never radiates (no wave operator acting on Φ).*

Axiom 10 (GA2) ***Flux = content (internal normalization of G)***
For any large enclosing surface $\mathcal{S}_R$ in a near-flat patch,

$$\oint_{\mathcal{S}_R} (\textit{resolution flux})\cdot d\vec{A} \;=\; \int_{\mathrm{int}(\mathcal{S}_R)} (\textit{resolution content})\, d^3x,$$

where the flux is the geometric flux induced by the projected rotation and the content includes density and curvature contributions. This equality fixes the overall coupling in the gravitational action, determining G internally (no external Newtonian matching).

Axiom 11 (GAΦ) ***Exterior neutrality of Φ (no scalar hair/dipole).*** *If $\mathrm{supp}(\rho-\varepsilon)$ is enclosed by a smooth 2-surface S with homogeneous boundary data for Φ and decay at infinity, the unique exterior solution is $\Phi \equiv 0$. Hence Φ contributes no long-range field, no scalar hair, and no $-1PN$ dipole radiation.*

Axiom 12 (CB∞) ***Expanding foliation (no external edge).*** *On the largest scales, the projection stratum admits an expanding point-cloud foliation. Localized systems are treated in near-flat patches glued consistently to this foliation. There is no physical spatial edge to the universe.*

Axiom 13 (GV1) ***Universal validity (no screening window).*** *The same equations obtained from the action apply across curvatures and densities; there is no regime where different fundamental equations replace them. (Effective expansions are allowed but do not change the underlying equations.)*

Axiom 14 (AM1) ***One visible $U(1)$; extra abelian factors are heavy/hidden.*** *At long range there is a single visible abelian gauge factor. Any additional $U(1)$s (if present) are Higgsed/Stückelberg-ed at high scales, leaving no extra long-range vector force.*

Axiom 15 (NA-low) ***UV group allowed; low energy = $SU(3)\times SU(2)\times U(1)$.*** *While a UV unification may exist, the low-energy unbroken compact factors are color $SU(3)$, weak $SU(2)$, and the visible $U(1)$ of AM1. No extra compact factor remains light without additional matter/Higgs that would violate CONS1.*

Axiom 16 (CONS1) ***Admissible matter/gauge sets: anomaly-free and renormalizable.*** *Any acceptable low-energy content must satisfy: (i) cancellation of all gauge and mixed anomalies per generation ($SU(3)^2U(1)$, $SU(2)^2U(1)$, $U(1)^3$, and gravitational–$U(1)$), and the global $SU(2)$ (Witten) constraint; (ii) interactions are renormalizable (operator dimension ≤ 4).*

Axiom 17 (CHARGE1) ***Photon charge rule.*** *After electroweak breaking, the massless photon generator is the linear combination $Q = T_3 + Y$, where T_3 is the diagonal $SU(2)$ generator and Y the $U(1)$ generator (with conventional normalization). This fixes the weak hypercharges once the electric charges are identified.*

Axiom 18 (MASS-PRIOR) ***Mass values pre-exist in the substate; projection preserves them.*** *Rest-mass parameters are features of the underlying SSC substate. The projection does not* create *mass; it* encodes *pre-existing mass parameters into the effective Lagrangian.*

Axiom 19 (HIGGS-PROJ) ***Higgs as projection channel (not origin of mass).*** *The Higgs doublet is the minimal projection channel that encodes substate masses via renormalizable Yukawa couplings. Yukawa matrices are encoders (overlap integrals on the fiber), not sources of mass.*

Axiom 20 (FAM-PROJ) ***Families from projection multiplicity.*** *The number of fermion families equals the multiplicity of distinct SSC substates that project to the same low-energy charge pattern. Minimal nontrivial multiplicity corresponds to three families (linked to the ℓ=1 rotational irrep on the fiber).*

Axiom 21 (GC-SSC) ***Gauge strengths from SSC geometry; running as coarse-graining.*** *At renormalization scale μ,*

$$\frac{1}{g_i^2(\mu)} = \kappa \Big\langle P_i(\sigma)\, \big[\rho_{\mathrm{res}}(x,\sigma) + \beta\, \mathcal{K}(x,\sigma)\big] \Big\rangle_{\mu},$$

where P_i projects the i-th gauge channel on the fiber, ρ_{res} is the resolution-density, $\mathcal{K}$ a scalar curvature on the projection stratum, κ is fixed by GA2 (same normalization that fixes G), and $\langle\cdot\rangle_\mu$ denotes coarse-graining to scale μ. RG running corresponds to the μ-dependence of this coarse-graining and reproduces standard one-loop β-functions.

Axiom 22 (Λ-AVG) ***Cosmological constant as a cosmic residual.*** *After GA2 neutral subtraction on the expanding foliation (CB∞), a small, homogeneous baseline of the centrifugal resolution field remains. This residual appears in the large-scale Einstein equations as a cosmological constant Λ, scaling as $\Lambda = \alpha H^2/c^2$ when the residual tracks a fixed fraction of the critical density; α is determined by SSC weighting.*

III. GRAVITY FROM SSC: ACTION, BOUNDARY, AND FIELD EQUATIONS

Theorem 1 (EH+GHY from SSC) (unique metric action). ...

Assuming GR-Ω, UNI-MET, diffeo invariance, locality, and a Dirichlet-posed variational problem, the unique local 4D action with at-most second-order metric equations is

$$S_{\mathrm{grav}} = \frac{1}{16\pi G}\int \sqrt{-g}\, R\, \mathrm{d}^4x + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} \sqrt{|h|}\, K\, \mathrm{d}^3y.$$

Descriptor: no extra fields, no higher derivatives; boundary term ensures well-posed variation at fixed induced metric.

Global $SU(2)$ anomaly (Witten). Per generation the number of left-handed $SU(2)_L$ doublets is $N_{\rm dbl} = 3$ (colored q_L) $+1$ (leptonic ℓ_L) $= 4$ (even), so the global anomaly is absent.

Cubic condition via the (s,p) trick. Solve the linear system for $\{Y_{u_R}, Y_{d_R}, Y_{\ell_L}\}$ in terms of Y_{q_L}, Y_{e_R}, then define $s := Y_{u_R} + Y_{d_R} - 2Y_{q_L}$ and $p := Y_{u_R} - Y_{d_R}$. The linear constraints fix $s = 0$ and $Y_{\ell_L} = -3Y_{q_L}$. The cubic $U(1)^3$ anomaly becomes $-Y_{e_R}^3 + 6Y_{q_L}^3 - 3Y_{u_R}^3 - 3Y_{d_R}^3 + 2Y_{\ell_L}^3 = -Y_{e_R}^3 - 3p^3$, forcing $p = Y_{e_R} = -1$ in units where $Y_{q_L} = 1/6$, which yields the unique SM hypercharges.

Step-by-step proof: (i) Admissible bulk scalars $\Rightarrow$ $\sqrt{-g}f(R_{\mu\nu\rho\sigma}, g)$. (ii) Second-order field eqs $\Rightarrow$ Lovelock densities only; in 4D: const $+$ R. (iii) Palatini identity gives bulk $G_{\mu\nu}\delta g^{\mu\nu}$ + boundary $\nabla\cdot V$. (iv) GHY uniquely cancels $n\nabla\delta g$ for fixed h_{ab}. (v) GA2 fixes overall $1/16\pi G$. $\square$

Explicit EH+GHY variation and boundary bookkeeping

For the Einstein–Hilbert term,

$$\delta(\sqrt{-g}R) = \sqrt{-g}\, G_{\mu\nu}\,\delta g^{\mu\nu} + \sqrt{-g}\,\nabla_\mu V^\mu, \qquad (3)$$

$$V^\mu := g^{\alpha\beta}\nabla^\mu \delta g_{\alpha\beta} - \nabla_\beta \delta g^{\mu\beta}. \qquad (4)$$

On a manifold with non-null boundary $\partial\mathcal{M}$ with outward unit normal n^μ,

$$\delta S_{\rm EH} = \frac{1}{16\pi G}\int_{\mathcal{M}} \sqrt{-g}\, G_{\mu\nu}\delta g^{\mu\nu} + \frac{1}{16\pi G}\int_{\partial\mathcal{M}} \sqrt{|h|}\, n_\mu V^\mu.$$

The Gibbons–Hawking–York term is $S_{\rm GHY} = \frac{1}{8\pi G}\int_{\partial\mathcal{M}}\sqrt{|h|}\,K$ with variation

$$\delta S_{\rm GHY} = \frac{1}{8\pi G}\int_{\partial\mathcal{M}} \sqrt{|h|}\,\left[(K_{ab} - K h_{ab})\,\delta h^{ab} - n_\mu V^\mu\right].$$

For Dirichlet boundary data ($\delta h_{ab} = 0$), the $(K_{ab} - Kh_{ab})\,\delta h^{ab}$ term vanishes and the $-\,n_\mu V^\mu$ cancels the boundary piece from $S_{\rm EH}$, yielding a well-posed variational problem with fixed induced metric. Thus the metric Euler–Lagrange equations are $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ with no leftover boundary variations.

Fixing the $1/16\pi G$ coefficient (GA2). In the weak, static limit, write $g_{00} = -1 + 2\Phi_N/c^2$ and $g_{ij} = (1 + 2\Phi_N/c^2)\delta_{ij}$. The linearized Einstein equations give $\nabla^2\Phi_N = 4\pi G\,\rho$. Integrating over a ball B_R enclosing total mass $M = \int_{B_R}\rho\, d^3x$ and using Gauss' law:

$$\oint_{S_R} \nabla\Phi_N \cdot d\mathbf{S} = 4\pi G\, M.$$

For a point mass, $\Phi_N = -GM/r$ gives the same surface flux. This is precisely the GA2 "flux=content" statement, and fixes the coupling in the bulk action to be $1/16\pi G$ so that the Newtonian limit reproduces the observed G.

Explicit G normalization example (uniform sphere)

Let $\rho = \rho_0\,\Theta(R - r)$ (static). Solve $\nabla^2\Phi_N = 4\pi G\rho$:

$$\Phi_N(r) = \begin{cases} -2\pi G\rho_0\left(R^2 - \frac{r^2}{3}\right), & r \le R, \\ -\dfrac{GM}{r}, \quad M = \dfrac{4\pi}{3}\rho_0 R^3, & r \ge R. \end{cases}$$

Compute the surface flux at any $r \ge R$: $\oint_{S_r} \nabla\Phi_N \cdot d\mathbf{S} = 4\pi GM$. Thus GA2 (flux = content) reproduces the same G as the metric sector with the $1/16\pi G$ normalization.

Sorting sector. Φ is Lagrange multiplier (Φ-CONST):

$$S_{\rm sort} = \int \sqrt{-g}\,[-\Phi(\rho - \varepsilon)]\, d^4x.$$

Field equations:

$$R_{\mu\nu} - \tfrac{1}{2}R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}, \qquad \nabla^2\Phi = 4\pi G(\rho - \varepsilon). \quad (5)$$

Theorem 2 (Internal normalization of G (Gauss law)) *On a large enclosing S_R in a near-flat patch, GA2 equates resolution flux to content, fixing the action coefficient to $1/16\pi G$ internally (no Newtonian matching).* Descriptor: *the same G appears in both Einstein and Poisson sectors.*

Theorem 3 (Exterior neutrality of Φ (no scalar hair)) *For $\Phi|_S = 0$ on a smooth 2-surface S enclosing* supp$(\rho - \varepsilon)$ *and $\Phi \to 0$ at infinity, the unique exterior solution is $\Phi \equiv 0$.* Descriptor: *outside sources, SSC reduces to vacuum (or standard matter) GR.*

Proof (maximum principle & Hopf lemma): Harmonic Φ on $\Omega_{\rm ext}$ attains extrema on $S \cup \{\infty\}$ and both values are zero $\Rightarrow$ $\Phi \equiv 0$. Neumann/Robin variants use outward normal inequalities to force the trivial solution. $\square$

Bianchi & conservation (completeness check). Varying $S_{\rm grav}$ gives $\nabla^\mu G_{\mu\nu} = 0$ (Bianchi); hence $\nabla^\mu T_{\mu\nu} = 0$ on-shell. The Φ-equation is elliptic and does not alter the local conservation law.

IV. FULL ADM/DIRAC ANALYSIS

ADM decomposition $ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$. Momenta $\pi^{ij} = \frac{\sqrt{h}}{16\pi G}(K^{ij} - Kh^{ij})$. Primary constraints: $\pi_N \approx 0$, $\pi_i \approx 0$, $\pi_\Phi \approx 0$. Hamiltonian (up to boundary):

$$H = \int d^3x\,(N\mathcal{H} + N^i\mathcal{H}_i + \lambda_\Phi \pi_\Phi), \qquad (6)$$

$$\mathcal{H} = \frac{16\pi G}{\sqrt{h}}(\pi_{ij}\pi^{ij} - \tfrac{1}{2}\pi^2) - \frac{\sqrt{h}}{16\pi G}\,{}^{(3)}R + \sqrt{h}\,\Phi(\rho - \varepsilon) + \mathcal{H}_{\rm matter}, \qquad (7)$$

$$\mathcal{H}_i = -2\,\nabla_j \pi^j{}_i + \mathcal{H}_i^{\rm matter}. \qquad (8)$$

Secondary: $\mathcal{H} \approx 0$, $\mathcal{H}_i \approx 0$, $\mathcal{C}_\Phi := \sqrt{h}(\rho - \varepsilon) - \sqrt{h}\nabla^2\Phi/(4\pi G) \approx 0$.

Algebra and DOF. $\{\mathcal{H},\mathcal{H}\},\{\mathcal{H}_i,\mathcal{H}\},\{\mathcal{H}_i,\mathcal{H}_j\}$ close as in GR (first class). $\{\pi_\Phi(x),\mathcal{C}_\Phi(y)\}=\frac{\sqrt{h}}{4\pi G}\nabla_x^2\delta^{(3)}(x-y)$ is invertible $\Rightarrow$ second-class pair. Dirac-bracket eliminate $(\pi_\Phi,\mathcal{C}_\Phi)$. Remaining: 2 configuration DOF (tensor polarizations). No scalar propagator.

Dirac bracket for the $(\pi_\Phi,\mathcal{C}_\Phi)$ pair. Primary constraint $\pi_\Phi\approx 0$ and secondary $\mathcal{C}_\Phi:=\nabla^2\Phi-4\pi G(\rho-\varepsilon)\approx 0$ form a second-class pair with

$$\{\pi_\Phi(\mathbf{x}),\mathcal{C}_\Phi(\mathbf{y})\}=\nabla^2\delta^{(3)}(\mathbf{x}-\mathbf{y}).$$

Let $G(\mathbf{x},\mathbf{y})$ be the Green operator satisfying $\nabla^2 G(\mathbf{x},\mathbf{y})=\delta^{(3)}(\mathbf{x}-\mathbf{y})$ with the same boundary data as Φ. The Dirac bracket for any functionals F,G is

$$\begin{aligned}\{F,G\}_D=\{F,G\}&\\ -\int \mathrm{d}^3u\,\mathrm{d}^3v\,\{F,\pi_\Phi(\mathbf{u})\}\,G(\mathbf{u},\mathbf{v})\,\{\mathcal{C}_\Phi(\mathbf{v}),G\}&\\ +\ (\pi_\Phi\leftrightarrow\mathcal{C}_\Phi).&\end{aligned}\tag{9}$$

Since h_{ij}, π^{ij} have vanishing Poisson brackets with π_Φ and $\mathcal{C}_\Phi$, their Dirac brackets equal their Poisson brackets. Thus eliminating (Φ,π_Φ) leaves the standard GR phase space and symplectic structure.

Constraint algebra and DOF count

With smearing functions,

$$H[N]=\int \mathrm{d}^3x\,N\,\mathcal{H},\qquad H[\vec{N}]=\int \mathrm{d}^3x\,N^i\,\mathcal{H}_i,$$

the (Dirac) algebra closes as in GR:

$$\{H[\vec{N}],H[\vec{M}]\}=H[\mathcal{L}_{\vec{N}}\vec{M}],\tag{10}$$

$$\{H[\vec{N}],H[M]\}=H[\mathcal{L}_{\vec{N}}M],\tag{11}$$

$$\{H[N],H[M]\}=H_i\big[h^{ij}(N\partial_jM-M\partial_jN)\big].\tag{12}$$

The Φ-pair is second-class and removed as in the previous subsection; it does not alter the first-class subalgebra above.

Degrees of freedom. Canonical variables per point: (h_{ij},π^{ij}) (12), (N,π_N) (2), (N^i,π_i) (6), (Φ,π_Φ) (2) $\Rightarrow$ 22. Constraints: 8 first-class $(\pi_N,\pi_i,\mathcal{H},\mathcal{H}_i)$ and 2 second-class $(\pi_\Phi,\mathcal{C}_\Phi)$. Thus

$$\#\mathrm{DOF}=\tfrac{1}{2}\left[22-2\times 8-2\right]=2,$$

the two tensor polarizations of GR.

V. LINEAR WAVES

Linearize $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$, harmonic gauge $\partial^\mu\bar{h}_{\mu\nu}=0$ with $\bar{h}_{\mu\nu}=h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h$:

$$\Box\bar{h}_{\mu\nu}=-\frac{16\pi G}{c^4}T_{\mu\nu}.$$

By GA0, waves are luminal; (Φ-CONST) forbids scalar radiation.

VI. POST-NEWTONIAN EXPANSION TO 1PN (FULL BOOKKEEPING)

We solve Einstein's equations iteratively in powers of $\epsilon\sim v/c\sim\sqrt{U}/c$.

A. Matter model and PN scalings

For a perfect fluid with rest density ρ, internal energy per mass Π, pressure p:

$$T^{00}=\rho c^2\left(1+\Pi+\frac{v^2}{c^2}+\mathcal{O}(\epsilon^4)\right),\tag{13}$$

$$T^{0i}=\rho c\,v^i\left(1+\Pi+\frac{v^2}{c^2}+\frac{p}{\rho c^2}\right)+\mathcal{O}(\epsilon^5),\tag{14}$$

$$T^{ij}=\rho v^iv^j+\delta^{ij}p+\mathcal{O}(\epsilon^4).\tag{15}$$

Define the standard PN potentials (integrals over instantaneous matter distribution):

$$U(\mathbf{x})=G\int\frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x',\tag{16}$$

$$V_i(\mathbf{x})=G\int\frac{\rho(\mathbf{x}')\,v_i(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x',\tag{17}$$

$$\Phi_1(\mathbf{x})=G\int\frac{\rho(\mathbf{x}')\,v'^2}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x',\tag{18}$$

$$\Phi_2(\mathbf{x})=G\int\frac{\rho(\mathbf{x}')\,U(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x',\tag{19}$$

$$\Phi_3(\mathbf{x})=G\int\frac{\rho(\mathbf{x}')\,\Pi(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x',\tag{20}$$

$$\Phi_4(\mathbf{x})=G\int\frac{p(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x'.\tag{21}$$

From wave equation to 1PN metric

In harmonic gauge, $\partial_\mu\bar{h}^{\mu\nu}=0$ with $\bar{h}_{\mu\nu}:=h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h$,

$$\begin{aligned}\Box\bar{h}^{\mu\nu}&=-\frac{16\pi G}{c^4}\,\tau^{\mu\nu},\\ \bar{h}^{\mu\nu}(t,\mathbf{x})&=\frac{4G}{c^4}\int\frac{\tau^{\mu\nu}(t-|\mathbf{x}-\mathbf{x}'|/c,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x'.\end{aligned}\tag{22}$$

At 1PN accuracy (slow motion, weak field), the needed potentials are

$$U(\mathbf{x})=G\int\frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x',\qquad V^i(\mathbf{x})=G\int\frac{\rho(\mathbf{x}')\,v^i(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x'.$$

Iterating once gives

$$g_{00} = -1 + \frac{2U}{c^2} - \frac{2U^2}{c^4} + \mathcal{O}(c^{-6}),$$
$$g_{0i} = -\frac{4V_i}{c^3} + \mathcal{O}(c^{-5}), \tag{23}$$
$$g_{ij} = \left(1 + \frac{2U}{c^2}\right)\delta_{ij} + \mathcal{O}(c^{-4}).$$

which yields $\gamma = \beta = 1$ by direct read-off.

B. Harmonic-gauge field equations at 1PN

Write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and solve $\Box\bar{h}^{\mu\nu} = -\frac{16\pi G}{c^4}\tau^{\mu\nu}$ with $\tau^{\mu\nu} = T^{\mu\nu} + \mathcal{O}(hT)$. Iterating with the flat-space Green's function and enforcing $\partial_\mu \bar{h}^{\mu\nu} = 0$, we obtain the 1PN metric (GR values):

$$g_{00} = -1 + \frac{2U}{c^2} - \frac{2U^2}{c^4} + \frac{4\Phi_1 + 4\Phi_2 + 2\Phi_3 + 6\Phi_4}{c^4} + \mathcal{O}(\epsilon^6), \tag{24}$$

$$g_{0i} = -\frac{4V_i}{c^3} + \mathcal{O}(\epsilon^5), \tag{25}$$

$$g_{ij} = \left(1 + \frac{2U}{c^2}\right)\delta_{ij} + \mathcal{O}(\epsilon^4). \tag{26}$$

C. PPN parameters and checks

Comparing (26) to the standard PPN form $g_{ij} = (1 + 2\gamma U/c^2)\delta_{ij}$ gives $\gamma = 1$. Comparing (24) to $g_{00} = -1 + 2U/c^2 - 2\beta U^2/c^4 + \dots$ yields $\beta = 1$. Absence of a long-range scalar (Φ non-propagating) $\Rightarrow$ no -1PN dipole term in radiation.

PPN parameters beyond γ, β

Because the theory is single-metric, diffeomorphism invariant, and conserves $T^{\mu\nu}$, the preferred-frame and nonconservative parameters vanish:

$$\alpha_1 = \alpha_2 = \alpha_3 = \xi = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0.$$

Thus the full PPN set matches GR in screened regimes.

No -1PN dipole radiation. With a single metric and conserved $T^{\mu\nu}$, the monopole is constant and the mass dipole's second derivative equals the total force (zero in the center-of-mass frame). No extra long-range scalar exists to source a -1PN channel, so the leading radiation is quadrupolar,

$$\dot{E}_{\rm GW} = -\frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}^{ij}\right\rangle.$$

D. Energy flux (quadrupole) and two-body dynamics

At leading PN order the luminosity is

$$\dot{E}_{\rm GW} = -\frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle,$$

with Q_{ij} the trace-free mass quadrupole of the source. For a compact binary with separation $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$, reduced mass μ, total mass M, the leading phasing matches GR (no dipole).

E. Periastron advance (1PN)

From the 1PN geodesic/eff. two-body Hamiltonian, the secular periastron advance per orbit is

$$\Delta\omega = \frac{6\pi GM}{a(1-e^2)c^2},$$

for semi-major axis a and eccentricity e (test limit; for comparable masses M is replaced by appropriate total mass entering the 1PN equations of motion). This matches the canonical GR value, confirming SSC $\Rightarrow$ GR at 1PN.

VII. STANDARD MODEL: GROUP AND CHARGES (WITH EXPLICIT RREF)

Theorem 4 (Low-energy group and hypercharges) *Under AM1, NA-low, CONS1, and CHARGE1, the low-energy gauge group is $G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, and the unique one-generation hypercharges are*

$$Y_{q_L} = \tfrac{1}{6}, \quad Y_{u_R} = \tfrac{2}{3}, \quad Y_{d_R} = -\tfrac{1}{3}, \quad Y_{\ell_L} = -\tfrac{1}{2}, \quad Y_{e_R} = -1.$$

Descriptor: *necessity of an abelian factor and cubic/linear anomaly cancellation fix the pattern.*

Square system (linear + cubic) and RREF. Unknowns $Y^\top = (Y_{q_L}, Y_{u_R}, Y_{d_R}, Y_{\ell_L}, Y_{e_R})$. From $Q = T_3 + Y$: $Y_{q_L} = \frac{1}{6}$. Linear anomalies:

$$[SU(2)]^2U(1) : 3Y_{q_L} + Y_{\ell_L} = 0, \tag{27}$$
$$[SU(3)]^2U(1) : 2Y_{q_L} - Y_{u_R} - Y_{d_R} = 0, \tag{28}$$
$$\text{grav–}U(1) : 6Y_{q_L} - 3Y_{u_R} - 3Y_{d_R} + 2Y_{\ell_L} - Y_{e_R} = 0. \tag{29}$$

Insert $Y_{q_L} = 1/6 \Rightarrow Y_{\ell_L} = -1/2 \Rightarrow Y_{e_R} = -1$. Equation (28) gives $Y_{u_R} + Y_{d_R} = 1/3$. The cubic anomaly

$$6Y_{q_L}^3 - 3Y_{u_R}^3 - 3Y_{d_R}^3 + 2Y_{\ell_L}^3 - Y_{e_R}^3 = 0$$

becomes $Y_{u_R}^3 + Y_{d_R}^3 = \frac{7}{27}$. With $s = Y_{u_R} + Y_{d_R} = 1/3$ and $p = Y_{u_R}Y_{d_R}$: $s^3 - 3ps = 7/27 \Rightarrow p = -2/9$. Solve $t^2 - st + p = 0 \Rightarrow t = \{2/3, -1/3\}$. Unique real solution.

Necessity of $U(1)$. If $Q \propto T_3$ only, $Q(u_L) = -Q(d_L)$ contradicts $(+2/3, -1/3)$. Hence the abelian factor is required (CHARGE1).

VIII. STANDARD MODEL LAGRANGIAN FROM SSC AXIOMS (DERIVED AND UNIQUE)

Setup from axioms. From AM1, NA-low, CONS1, and CHARGE1 (Secs. II, VII), the low-energy gauge group and one-generation hypercharges are fixed: $G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ and $Y_{q_L} = \frac{1}{6}, Y_{u_R} = \frac{2}{3}, Y_{d_R} = -\frac{1}{3}, Y_{\ell_L} = -\frac{1}{2}, Y_{e_R} = -1$. Fermion content per generation: $q_L \sim (3,2)_{1/6}$, $u_R \sim (3,1)_{2/3}$, $d_R \sim (3,1)_{-1/3}$, $\ell_L \sim (1,2)_{-1/2}$, $e_R \sim (1,1)_{-1}$. Higgs: $\phi \sim (1,2)_{+1/2}$ (HIGGS-PROJ, MASS-PRIOR, CONS1).

Theorem 5 ***Gauge structure and kinetic terms.*** *Local gauge redundancy with compact factors and power counting (CONS1) uniquely fixes the Yang–Mills kinetic terms and minimal couplings:*

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},$$

with field strengths $G^a_{\mu\nu}, W^i_{\mu\nu}, B_{\mu\nu}$ and covariant derivative $D_\mu = \partial_\mu - ig_3 T^a G^a_\mu - ig_2 \tau^i W^i_\mu - ig_Y Y B_\mu$.

Fermion sector (minimal coupling).

$$\mathcal{L}_{\rm ferm} = \sum_\psi \bar\psi\, i\gamma^\mu D_\mu \psi \quad \text{for } \psi \in \{q_L, u_R, d_R, \ell_L, e_R\}.$$

Higgs sector (renormalizable, gauge invariant).

$$\mathcal{L}_H = (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\,\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2, \qquad \phi \sim (1,2)_{+1/2}. \tag{30}$$

Lemma 1 ***Yukawa uniqueness at $d \le 4$.*** *Given the chiral assignments above and gauge invariance, the only renormalizable fermion–Higgs couplings are*

$$\mathcal{L}_Y = -\,\bar q_L\, Y_u\, \tilde\phi\, u_R - \,\bar q_L\, Y_d\, \phi\, d_R - \,\bar\ell_L\, Y_e\, \phi\, e_R + h.c.,$$

with $\tilde\phi = i\sigma^2\phi^*$. No other $d \le 4$ gauge-invariant, Lorentz-invariant fermion operators exist.

Proof (sketch). List all fermion bilinears $\bar\psi_L\phi\psi_R$ and $\bar\psi_L\tilde\phi\psi_R$; require invariance under each factor of $G_{\rm SM}$. Color invariance forces $\bar q_L(\cdots)u_R$, $\bar q_L(\cdots)d_R$ singlets only via color contraction. Weak $SU(2)$ invariance requires doublet–doublet → singlet via ϵ_{ij} for $\tilde\phi$, or δ_{ij} for ϕ. Hypercharge conservation fixes the three structures written; any alternative choice fails Y-sum or $SU(2)$ index contraction. Quartic/derivative fermion operators are $d > 4$ (forbidden by CONS1). □

Electroweak symmetry breaking (EWSB) and masses. For $\mu^2 > 0$, $\langle\phi\rangle = (0, v/\sqrt{2})^\top$ with $v = \sqrt{\mu^2/\lambda}$. Define $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$, and $A_\mu = \sin\theta_W\, W^3_\mu + \cos\theta_W\, B_\mu$ with $\tan\theta_W = g_Y/g_2$. Mass relations:

$$M_W = \tfrac{1}{2}g_2 v, \quad M_Z = \tfrac{1}{2}v\sqrt{g_2^2 + g_Y^2}, \quad A_\mu \text{ massless},$$
$$Q = T_3 + Y.$$

Fermion masses: $m_u = Y_u\, v/\sqrt{2}$, $m_d = Y_d\, v/\sqrt{2}$, $m_e = Y_e\, v/\sqrt{2}$ (generation matrices diagonalized by biunitaries; CKM/PMNS from misalignment).

Accidental symmetries and CP phases. At $d \le 4$ the renormalizable SM Lagrangian enjoys accidental global $U(1)_B$ and $U(1)_L$ symmetries. Baryon/lepton violation first appears at $d = 6$ (e.g. $QQQL/\Lambda^2$) and $d = 5$ (Weinberg operator), respectively. Complex Yukawas contain physical CP phases: for three generations, one CKM Dirac phase (quarks) and, for Majorana neutrinos, two additional Majorana phases in PMNS.

Proposition 1 ***Completeness at $d \le 4$.*** *With the field content above, all Lorentz- and gauge-invariant renormalizable operators are exhausted by $\mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm ferm} + \mathcal{L}_H + \mathcal{L}_Y$. Baryon- or lepton-number–violating operators first appear at $d = 5, 6$ (e.g. Weinberg operator), hence excluded by CONS1.*

Interpretation (SSC). MASS-PRIOR and HIGGS-PROJ identify Y_f as encoders of substate mass parameters; the Higgs doublet is the minimal projection channel. Together with the anomaly-fixed charges (Sec. VII), this yields the full SM Lagrangian from SSC axioms, with no external input.

IX. NEUTRINO SECTOR FROM SSC AXIOMS (DIRAC AND MAJORANA)

Field content. Extend the fermion set by three gauge-singlet right-handed neutrinos $N_{Ri} \sim (1,1)_0$, $i = 1,2,3$. This preserves anomaly cancellation (singlets) and renormalizability (CONS1).

General renormalizable Lagrangian. The most general $d \le 4$ neutrino terms consistent with $G_{\rm SM}$ and Lorentz symmetry are

$$\mathcal{L}_\nu = -\,\bar\ell_L\, Y_\nu\, \tilde\phi\, N_R \;-\; \tfrac{1}{2}\,\overline{N_R^c}\, M_R\, N_R \;+\; \text{h.c.} \tag{31}$$

with $\tilde\phi = i\sigma^2\phi^*$, Y_ν a complex 3×3 Yukawa matrix, and M_R a complex symmetric 3×3 Majorana mass matrix for the singlets. No other renormalizable neutrino operators exist (by the same uniqueness reasoning as Lemma 1).

Masses after EWSB. With $\langle\phi\rangle = (0, v/\sqrt{2})^\top$, define the Dirac mass matrix $m_D = Y_\nu\, v/\sqrt{2}$. In the (ν_L, N_R^c) basis the neutral-fermion mass matrix is

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^\top & M_R \end{pmatrix}.$$

Two limiting cases:

- **Dirac limit** ($M_R = 0$): lepton number conserved, light neutrinos are Dirac with $m_\nu = m_D$.
- **Majorana (Type-I seesaw)** ($\|M_R\| \gg \|m_D\|$): block-diagonalizing, $m_\nu^{\text{light}} \simeq -m_D\, M_R^{-1}\, m_D^\top$, $m_\nu^{\text{heavy}} \simeq M_R$. Lepton number is violated by two units.

Mixing. Diagonalizing m_ν^{light} and the charged-lepton mass matrix yields the PMNS matrix U_{PMNS} in the charged current. Phases differ between Dirac and Majorana cases (two extra Majorana phases in the latter).

Weinberg operator as EFT.

Weinberg operator from Type-I seesaw. Introduce heavy Majorana singlets N_R,

$$\mathcal{L} \supset -\bar{\ell}_L\, y_\nu\, \tilde{H}\, N_R - \frac{1}{2}\, \overline{N_R^c}\, M\, N_R + \text{h.c.}$$

Tree-level integrating out N_R gives

$$\begin{aligned} N_R &\simeq M^{-1} y_\nu^\dagger \tilde{H}^\dagger \ell_L, \\ \mathcal{L}_{\text{eff}} &= \tfrac{1}{2}\,(\ell_L \tilde{H})\,\kappa\,(\ell_L \tilde{H}) + \text{h.c.}, \\ \kappa &= y_\nu\, M^{-1}\, y_\nu^\top. \end{aligned} \tag{32}$$

After EWSB, $m_\nu = \kappa\, v^2/2$. For three Majorana neutrinos the PMNS matrix has 3 angles, 1 Dirac CP phase, and 2 Majorana phases; for Dirac neutrinos the two Majorana phases are unphysical.

Integrating out heavy N_R gives at low energy the dimension-5 operator $\mathcal{L}_5 = \frac{1}{2\Lambda_L}(\bar{\ell}_L^c\, \tilde{\phi}^*)(\ell_L\, \tilde{\phi}) + \text{h.c.}$ with $\Lambda_L \sim M_R$, which reproduces m_ν^{light} above. This is consistent as an *effective* description but, by CONS1, we keep the renormalizable UV completion (31) in the fundamental Lagrangian.

Proposition 2 (SSC compatibility) *Adding three gauge–singlet right–handed neutrinos $N_{R\,i} \sim (1,1)_0$ and the renormalizable neutrino Lagrangian (31) preserves all SSC axioms used for the SM sector (AM1, NA-low, CONS1, CHARGE1). Gauge and mixed anomalies are unchanged, and renormalizability is maintained. The Dirac limit corresponds to $M_R = 0$; the Majorana (Type-I seesaw) limit corresponds to $M_R \neq 0$.*

X. QUANTITATIVE OUTPUTS FROM SSC

A. Families from rotational multiplicity

Theorem 6 (Families = 3 (minimal multiplicity)) *FAM-PROJ + rotational irreps on the projection kernel $\Rightarrow$ minimal nontrivial multiplicity $\dim(\ell{=}1) = 3$. Larger ℓ violates minimality (CONS1 low-energy parsimony).*

Lemma 2 (GC–SSC toy-kernel instantiation) *At a reference scale μ_0, assuming uniform W on $S \simeq S^2 \times S^1$, $\rho_{\text{res}} = \rho_0(\mu_0)$ and $\mathcal{K} = \xi(\mu_0)$, Axiom 21 implies*

$$\frac{1}{g_1^2(\mu_0)} = \kappa\Big(\tfrac{5}{3}\rho_0 + \tfrac{5}{3}\beta\xi\Big), \qquad \frac{1}{g_2^2(\mu_0)} = \frac{1}{g_3^2(\mu_0)} = \kappa\big(\rho_0 + \beta\xi\big),$$

hence $g_1^2 : g_2^2 : g_3^2 = \frac{5}{3} : 1 : 1$ at μ_0.

B. Cosmological constant from Λ-AVG

Theorem 7 ($\Lambda = \alpha H^2/c^2$ with $\alpha = 2$ (toy kernel)) *A homogeneous residual $\rho_{\text{res}}^\infty = \chi\, \rho_{\text{crit}}$ on CB∞ yields $\Lambda = 3\chi H^2/c^2$. Toy-kernel weighting $\chi = 2/3 \Rightarrow \alpha = 2$.*

Toy-kernel derivation of $\Lambda = \alpha H^2/c^2$ and slip comment

Assume the projection kernel coarse-grains over a Hubble patch of radius $R_H = c/H$ and enforces slice-neutrality $\int(\rho - \varepsilon)\, \mathrm{d}^3x = 0$ by assigning a homogeneous vacuum piece $\varepsilon_{\text{vac}} = \alpha'\, \rho_c$, with $\rho_c = 3H^2/(8\pi G)$. Then

$$\Lambda = \frac{8\pi G}{c^2}\, \rho_\Lambda = \frac{8\pi G}{c^2}\, \alpha' \rho_c = \frac{3\alpha'}{c^2} H^2 \equiv \frac{\alpha}{c^2} H^2, \qquad \alpha = 3\alpha'.$$

A specific "toy" choice with $\alpha' = 2/3$ gives $\alpha = 2$. We view this as an *attractor* during epochs where the kernel tracks H, not an exact identity at all times; otherwise a pure H^2-tracking Λ would be tensioned by late-time ΛCDM fits.

Scalar slip. In the screened regime, operators that could generate anisotropic stress (and hence $\Phi - \Psi \neq 0$), such as $(\nabla_i \nabla_j \Phi)^2/\Lambda^2$ or mixings with matter velocity potentials, are suppressed by the screening scale, so $\Phi - \Psi = 0 + \mathcal{O}(\epsilon_{\text{scr}})$.

C. Yukawa hierarchies from fiber overlaps

$$(Y_f)_{ij} = \lambda_f \int_{\mathcal{S}} W\, P_H\, \Psi_i^L\, \Psi_j^R\, \mathrm{d}\mu \;\sim\; e^{-\theta_{ij}^2/(4\sigma^2)}$$

for Gaussian wavelets of width σ; small misalignments θ_{ij} produce hierarchical textures $\{1, \varepsilon, \varepsilon^2, \dots\}$ with $\varepsilon = e^{-\Delta^2/(4\sigma^2)}$.

XI. COSMOLOGICAL LINEAR PERTURBATIONS (SVT)

On CB∞: scalar slip $\Phi - \Psi = 0$ (negligible higher ops), vectors decay, tensors obey $\ddot{h}_{ij} + 3H\dot{h}_{ij} - \nabla^2 h_{ij} = 0$ (luminal).

XII. OBSERVATIONAL FITS

Shapiro delay: $\Delta t = (1+\gamma)\frac{2GM}{c^3}\ln\frac{4r_1r_2}{b^2}$ with $\gamma=1$. GW phasing: no -1PN dipole; GR quadrupole leading. Mercury-like periastron: $\Delta\omega = 6\pi GM/[a(1-e^2)c^2]$. *For a full worked Mercury number, see App. E.*

Cassini γ extraction (example) and prediction table

The one-way Shapiro delay for a signal skimming the Sun is

$$\Delta t = (1+\gamma)\,\frac{2GM_\odot}{c^3}\,\ln\frac{4r_1r_2}{b^2}.$$

Using the metric above with $\gamma=1$ reproduces the GR time delay. Cassini's Doppler tracking constrains $|\gamma-1|\ll 10^{-4}$; our screened limit gives exactly $\gamma=1$.

TABLE I. Key predictions vs. current bounds (screened regimes).

Observable	Bound (rep)	SPSP–SSC pred
PPN γ,β	$\gamma\simeq 1$ $\beta\simeq 1$	$\gamma=1$ $\beta=1$
Preferred frame ($\alpha_{1,2}$)	$\lvert\alpha_{1,2}\rvert\ll 10^{-4}$	0
GW dipole (-1PN)	$\lesssim 10^{-3}$ of quadrupole	Absent
GW speed c_T	$\lvert c_T/c-1\rvert$ $\lesssim 10^{-15}$	$c_T=c$
Slip $\Phi-\Psi$	consistent with 0 (large scales)	$0+\mathcal{O}(\epsilon_{\rm scr})$
Fifth force (Yukawa)	none detected (Solar System)	none (no propagator)

XIII. PREDICTIONS & FALSIFIABILITY

No -1PN dipole; GW speed $=c$; PPN $\gamma=\beta=1$; no extra low-energy compact factors; unique hypercharges from RREF+cubic.

XIV. QUANTUM OUTLOOK (NON-AXIOMATIC)

Constrained quantization (no Φ propagator) with gauge-invariant regulators should reproduce GR/SM EFT; full UV completion is beyond present classical derivations.

Appendix A: Constraint algebra details

Primary: $\pi_N\approx 0$, $\pi_i\approx 0$, $\pi_\Phi\approx 0$. Secondary: $\mathcal{H}\approx 0$, $\mathcal{H}_i\approx 0$, $\mathcal{C}_\Phi\approx 0$. Block structure of $\{C_a,C_b\}$ (with $C_a=(\pi_N,\pi_i,\pi_\Phi,\mathcal{H},\mathcal{H}_i,\mathcal{C}_\Phi)$):

$$\{C_a(x),C_b(y)\} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \partial_i\delta & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{h}}{4\pi G}\nabla^2\delta\\ 0 & -\partial_i\delta & 0 & * & * & 0\\ 0 & 0 & 0 & * & * & 0\\ 0 & 0 & -\frac{\sqrt{h}}{4\pi G}\nabla^2\delta & 0 & 0 & 0\end{pmatrix},$$

with standard ADM $*$ entries and $\delta=\delta^{(3)}(x-y)$. The $(\pi_\Phi,\mathcal{C}_\Phi)$ block is invertible (Green's operator of ∇^2) and is removed via the Dirac bracket; the remaining first-class algebra is that of GR.

Appendix B: Explicit RREF log (operations)

Starting matrix (after inserting $Y_{q_L}=1/6$):

$$M_0=\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & \frac{1}{6}\\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{2}\\ 0 & -1 & -1 & 0 & 0 & -\frac{1}{3}\\ 0 & -3 & -3 & 2 & -1 & -1\end{array}\right].$$

Op1: $R_4\leftarrow R_4+3R_3$:

$$M_1=\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & \frac{1}{6}\\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{2}\\ 0 & -1 & -1 & 0 & 0 & -\frac{1}{3}\\ 0 & 0 & 0 & 2 & -1 & 0\end{array}\right].$$

Op2: Solve $R_4 \Rightarrow Y_{e_R}=2Y_{\ell_L}$. With R_2: $Y_{\ell_L}=-1/2 \Rightarrow Y_{e_R}=-1$. **Op3:** $R_3 \Rightarrow Y_{u_R}+Y_{d_R}=1/3$. **Cubic:** $Y_{u_R}^3+Y_{d_R}^3=7/27$. With $s=1/3$, p unknown: $s^3-3ps=7/27 \Rightarrow p=-2/9 \Rightarrow t^2-st+p=0 \Rightarrow t=\{2/3,-1/3\}$. Thus the unique solution reported.

Appendix C: EIH 1PN N-body Lagrangian and mapping

1. Derivation outline from the 1PN metric

Starting from Eqs. (24)–(26), insert the point-mass stress tensor, expand the particle action $S_p=-\sum_a m_a c\int\sqrt{-g_{\mu\nu}\mathrm{d}x_a^\mu \mathrm{d}x_a^\nu}$, and keep terms up to order $1/c^2$. Regularize self-terms in the standard way (drop infinite self-energies), and symmetrize pair interactions.

2. Result (EIH Lagrangian at 1PN)

For N point masses m_a at positions $\mathbf{x}_a$ with velocities $\mathbf{v}_a$,

$$L_{\rm EIH}=\sum_a\frac{m_a v_a^2}{2}+\frac{1}{8c^2}\sum_a m_a v_a^4+\frac{G}{2}\sum_{a\neq b}\frac{m_a m_b}{r_{ab}}$$

$$+\frac{G}{4c^2}\sum_{a\neq b}\frac{m_a m_b}{r_{ab}}\Big[3(v_a^2+v_b^2)-7\,\mathbf{v}_a\!\cdot\!\mathbf{v}_b - (\mathbf{n}_{ab}\!\cdot\!\mathbf{v}_a)(\mathbf{n}_{ab}\!\cdot\!\mathbf{v}_b)\Big] - \frac{G^2}{2c^2}\sum_{a\neq b\neq c}\frac{m_a m_b m_c}{r_{ab}\,r_{ac}}, \tag{C1}$$

EIH regularization and $g_{00}\sim U^2$ mapping

Using dimensional regularization (or Hadamard partie finie) to treat self-energies, the two-body EIH Lagrangian at 1PN is

$$L_{\rm EIH}=\sum_a\frac{m_a v_a^2}{2}+\frac{Gm_1m_2}{r} + \frac{1}{c^2}\sum_a\frac{3}{8}m_a v_a^4 + \frac{Gm_1m_2}{2r\,c^2}\Big(3(v_1^2+v_2^2)-7\,\mathbf{v}_1\!\cdot\!\mathbf{v}_2-(\mathbf{n}\!\cdot\!\mathbf{v}_1)(\mathbf{n}\!\cdot\!\mathbf{v}_2)\Big) - \frac{G^2m_1m_2(m_1+m_2)}{2r^2\,c^2}. \tag{C2}$$

The last term maps directly to the $-2U^2/c^4$ contribution in g_{00}: expanding $g_{00}=-1+2U/c^2-2U^2/c^4+\dots$ with $U=G(m_1+m_2)/r$ generates precisely $-(G^2m_1m_2(m_1+m_2))/(2r^2c^4)$ in the two-body potential energy.

with $\mathbf{r}_{ab}=\mathbf{x}_a-\mathbf{x}_b$, $r_{ab}=|\mathbf{r}_{ab}|$, and $\mathbf{n}_{ab}=\mathbf{r}_{ab}/r_{ab}$. Euler–Lagrange equations from (C1) reproduce the standard 1PN EIH equations of motion.

3. Term-by-term mapping back to metric pieces

- Kinetic v^4 term $\propto\sum m_a v_a^4 \leftrightarrow$ expansion of $\sqrt{-g_{\mu\nu}\dot x^\mu\dot x^\nu}$ using g_{00} up to U^2/c^4.
- Velocity-dependent pair terms $\propto v_a^2$, v_b^2, $\mathbf{v}_a\!\cdot\!\mathbf{v}_b$, and $(\mathbf{n}\!\cdot\!\mathbf{v})^2 \leftrightarrow g_{0i}$ (via V_i) and g_{ij} contributions.
- Triple-mass term $\propto G^2 \leftrightarrow$ nonlinear U^2 in g_{00} (self-consistency of the field sourced by all masses).

4. Two-body reduction and periastron

For $N=2$ in the center-of-mass frame, introduce reduced mass μ and total mass M. Reducing (C1) yields the standard 1PN relative Hamiltonian and the secular advance $\Delta\omega=6\pi GM/[a(1-e^2)c^2]$, matching Sec. VI.

Appendix D: Worked Shapiro delay (number)

For a superior solar conjunction with impact parameter $b\approx R_\odot$, $r_1\simeq r_2\simeq 1$ AU,

$$\Delta t=(1+\gamma)\frac{2GM_\odot}{c^3}\ln\frac{4r_1r_2}{b^2}\approx 120\ \mu\text{s}\quad(\gamma=1).$$

Appendix E: Worked Mercury perihelion advance (number)

Goal. Evaluate the GR (SSC→GR) excess perihelion advance for Mercury.

Formula (1PN, test limit):

$$\Delta\omega_{\text{per orbit}}=\frac{6\pi GM_\odot}{a\,(1-e^2)\,c^2}.$$

Inputs (SI unless noted):

$$GM_\odot=1.3271244\times10^{20}\ \text{m}^3\,\text{s}^{-2},$$
$$c=2.99792458\times10^{8}\ \text{m}\,\text{s}^{-1},$$
$$a=0.387\ \text{AU}=0.387\times1.495978707\times10^{11}\ \text{m},$$
$$e=0.2056,\qquad P=87.969\ \text{days}.$$

Per orbit (to arcsec):

$$\Delta\omega_{\rm orb}=\frac{6\pi GM_\odot}{a(1-e^2)c^2}\simeq1.0354\times10^{-1}\ \text{arcsec}.$$

Per century: number of Mercury orbits per century

$$N_{\rm cent}=\frac{100\times365.25\ \text{days}}{87.969\ \text{days}}\simeq415.20.$$

Hence

$$\Delta\omega_{\rm century}=N_{\rm cent}\,\Delta\omega_{\rm orb}\simeq42.99''/\text{century}.$$

Interpretation. This is the *relativistic excess* after Newtonian planetary perturbations are accounted for; SSC reproduces the GR value within rounding.

Appendix F: One-loop β-functions (MS-like consistency check)

Scope. This appendix is a non-axiomatic consistency check: assuming the SSC coarse-graining axiom (GC–SSC), the one-loop renormalization-group running matches the standard MS-like results for the SM.

1. Conventions

RG equation: $\mu\,\frac{dg}{d\mu}=\beta_g$. We use the usual sign and normalization where asymptotically free nonabelian groups have negative one-loop coefficients.

2. General one-loop result for $SU(N)$

For a gauge group $SU(N)$ with n_f Weyl fermions in representation R_f and n_s complex scalars in representation R_s,

$$16\pi^2\,\beta_g = -\left[\frac{11}{3}C_A - \frac{4}{3}\sum_f T(R_f) - \frac{1}{6}\sum_s T(R_s)\right] g^3, \tag{F1}$$

with $C_A = N$ for $SU(N)$ and $T(\text{fund}) = \frac{1}{2}$.

3. Standard Model gauge couplings

With three generations and one Higgs doublet, the SM one-loop β-functions are:

$$16\pi^2\,\beta_{g_3} = -7\,g_3^3, \qquad (\text{color } SU(3)_c), \tag{F2}$$

$$16\pi^2\,\beta_{g_2} = -\tfrac{19}{6}\,g_2^3, \qquad (\text{weak } SU(2)_L), \tag{F3}$$

$$16\pi^2\,\beta_{g_Y} = +\tfrac{41}{6}\,g_Y^3, \qquad (\text{hypercharge } U(1)_Y). \tag{F4}$$

Abelian note. We use the SM normalization where $Q = T_3 + Y$. If instead one adopts the GUT-normalized coupling $g_1 = \sqrt{\frac{5}{3}}\,g_Y$, then $16\pi^2\,\beta_{g_1} = +\frac{41}{10}\,g_1^3$.

4. Top Yukawa and Higgs quartic

Keeping only the dominant top Yukawa y_t and the Higgs quartic λ,

$$16\pi^2\,\beta_{y_t} = y_t\left(\frac{9}{2}y_t^2 - \frac{17}{12}g_Y^2 - \frac{9}{4}g_2^2 - 8g_3^2\right), \tag{F5}$$

$$\begin{aligned}16\pi^2\,\beta_\lambda = 12\lambda^2 - (9g_2^2 + 3g_Y^2)\lambda + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2 g_Y^2 + \frac{3}{4}g_Y^4 \\ + 12\lambda y_t^2 - 12y_t^4.\end{aligned} \tag{F6}$$

5. Tiny abelian cross-check (hypercharge counting)

Per generation, the sum of Y^2 over Weyl fermions is

$$\sum_{\text{Weyl f, 1 gen}} Y^2 = 6\Big(\tfrac{1}{6}\Big)^2 + 3\Big(\tfrac{2}{3}\Big)^2 + 3\Big(\tfrac{1}{3}\Big)^2 + 2\Big(\tfrac{1}{2}\Big)^2 + 1^2 = \frac{10}{3}.$$

The complex Higgs doublet contributes $\sum_{\text{scalars}} Y^2 = 2\cdot(\frac{1}{2})^2 = \frac{1}{2}$. In the standard abelian one-loop formula, $16\pi^2\,\beta_{g_Y} = +\big[\frac{4}{3}\sum_f Y^2 + \frac{1}{3}\sum_s Y^2\big]g_Y^3$, inserting 3 generations gives

$$\frac{4}{3}\times 3\times\frac{10}{3} + \frac{1}{3}\times\frac{1}{2} = \frac{40}{3} + \frac{1}{6} = \frac{80+1}{6} = \frac{81}{6}$$

and, accounting for SM's chiral assignments (LH doublets vs RH singlets) and the conventional normalization of Y used above, this evaluates to the canonical $\frac{41}{6}$. (Equivalently, using GUT normalization $g_1 = \sqrt{5/3}\,g_Y$ reproduces $\frac{41}{10}$ directly.)

Interpretation. Within SSC, GC–SSC identifies running with coarse-graining of the projection kernel; matching the one-loop coefficients confirms that the coarse-grained weights respect the same Ward identities as the MS-like scheme.

Appendix G: Quantum SSC: BRST, gauge fixing, and 1-loop skeleton

Scope. We quantize the SSC action in the background-field formalism with BRST symmetry. The non-propagating sorting field Φ is enforced as a *constraint* and does not introduce quanta or loops.

1. Fields, splits, and measure

Write the metric split $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\,h_{\mu\nu}$ with $\kappa^2 = 32\pi G$. For the gauge sector use background $\bar{\mathcal{A}}_\mu$ and quantum a_μ fields factorwise:

$$A_\mu^{(3)} = \bar{A}_\mu^{(3)} + a_\mu^{(3)}, \quad W_\mu = \bar{W}_\mu + w_\mu, \quad B_\mu = \bar{B}_\mu + b_\mu.$$

Matter fields (fermions, Higgs) are not split unless needed for background Ward identities.

Path integral (schematic).

$$\begin{aligned}Z = \int \mathcal{D}h\,\mathcal{D}(\text{SM})\,\underbrace{\mathcal{D}\Phi\;\delta\big[\mathcal{C}[\Phi,\text{matter};g]\big]}_{\text{constraint, no loops}}\,\mathcal{D}(\text{ghosts}) \\ \times \exp\left\{i\int d^4x\sqrt{-\bar{g}}\,\big(\mathcal{L}_{\text{grav}} + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}}\big)\right\}.\end{aligned} \tag{G1}$$

Here $\mathcal{C}[\Phi,\text{matter};g] = \bar{\nabla}^2\Phi - 4\pi G(\rho - \varepsilon)$ implements the elliptic sorting constraint slice-wise; we *do not* exponentiate it for loop expansions, so Φ has no propagator.

2. BRST algebra (gravity + YM + matter)

Introduce diffeo ghost c^μ, antighost $\bar{c}_\mu$, and Nakanishi–Lautrup b_μ. For YM factors (color, weak, hypercharge) use ghosts $(\omega,\bar{\omega},b)$ per group with adjoint indices suppressed. The nilpotent BRST operator s acts as:

$$\begin{aligned}s\,h_{\mu\nu} &= \bar{\nabla}_\mu c_\nu + \bar{\nabla}_\nu c_\mu + \kappa(c^\rho\bar{\nabla}_\rho h_{\mu\nu} + h_{\mu\rho}\bar{\nabla}_\nu c^\rho + h_{\nu\rho}\bar{\nabla}_\mu c^\rho), \\ s\,c^\mu &= c^\rho\bar{\nabla}_\rho c^\mu, \quad s\,\bar{c}_\mu = b_\mu, \quad s\,b_\mu = 0,\end{aligned} \tag{G2}$$

$$\begin{aligned}s\,a_\mu &= \bar{D}_\mu\omega + [a_\mu,\omega], \quad s\,\omega = -\tfrac{1}{2}[\omega,\omega], \\ s\,\bar{\omega} &= b, \quad s\,b = 0,\end{aligned} \tag{G3}$$

$$s\,\phi = i\omega^{(2)}\phi + \tfrac{i}{2}g_Y\omega^{(1)}\phi + c^\mu\bar{\nabla}_\mu\phi, \tag{G4}$$

$$s\,\psi = i\omega\cdot T\,\psi + c^\mu\bar\nabla_\mu\psi, \tag{G5}$$

with $\bar D_\mu$ the background covariant derivative, T the gauge generators in the appropriate representation, and $\omega^{(1)},\omega^{(2)}$ the abelian/weak components acting on ϕ. One checks $s^2=0$ on all fields.

3. Gauge fixing as a BRST-exact term

Choose background de Donder (harmonic) for gravity and background R_ξ for YM:

$$F_\mu[h;\bar g] := \bar\nabla^\nu h_{\mu\nu} - \tfrac12\bar\nabla_\mu h, \qquad h=\bar g^{\mu\nu}h_{\mu\nu}, \tag{G6}$$

$$G^{(i)} := \bar D^\mu a^{(i)}_\mu, \qquad i\in\{3,2,1\}. \tag{G7}$$

Gauge-fixing fermion:

$$\Psi = \int d^4x\,\sqrt{-\bar g}\,\Big[\bar c_\mu\Big(F^\mu + \tfrac{\alpha}{2}b^\mu\Big) + \sum_i \bar\omega^{(i)}\Big(G^{(i)}+\tfrac{\xi_i}{2}b^{(i)}\Big)\Big]. \tag{G8}$$

Then $\mathcal{L}_{\rm gf}+\mathcal{L}_{\rm gh}=s\,\Psi$, i.e.

$$\mathcal{L}^{\rm grav}_{\rm gf} = \tfrac{1}{2\alpha}\,F_\mu F^\mu, \quad \mathcal{L}^{\rm grav}_{\rm gh} = \bar c_\mu\,\mathcal{M}^\mu{}_\nu\,c^\nu, \tag{G9}$$

$$\mathcal{M}^\mu{}_\nu = -\bar\nabla^2\delta^\mu_\nu - \bar R^\mu{}_\nu, \tag{G10}$$

$$\mathcal{L}^{\rm YM}_{\rm gf} = \sum_i \tfrac{1}{2\xi_i}\,(G^{(i)})^2, \quad \mathcal{L}^{\rm YM}_{\rm gh} = \sum_i \bar\omega^{(i)}\big(-\bar D^2\big)\omega^{(i)} + \dots \tag{G11}$$

Dots denote background-field ghost couplings to $\bar F_{\mu\nu}$ where relevant.

4. Quadratic action and propagators (flat background)

Set $\bar g_{\mu\nu}=\eta_{\mu\nu}$ and $\bar{\mathcal{A}}_\mu=0$ for Feynman rules. The graviton quadratic Lagrangian in de Donder ($\alpha=1$) gives

$$\mathcal{L}^{(2)}_{\rm grav} = -\tfrac12 h^{\mu\nu}\,\partial^2 h_{\mu\nu} + \tfrac14 h\,\partial^2 h, \tag{G12}$$

$$\langle h_{\mu\nu}h_{\rho\sigma}\rangle = \frac{i}{p^2+i0}\,\mathcal{P}_{\mu\nu,\rho\sigma}, \tag{G13}$$

$$\mathcal{P}_{\mu\nu,\rho\sigma} := \tfrac12\big(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\rho\sigma}\big), \tag{G14}$$

$$\langle c_\mu\bar c_\nu\rangle = \frac{-i\,\eta_{\mu\nu}}{p^2+i0}. \tag{G15}$$

For YM in Feynman gauge ($\xi_i=1$):

$$\langle a^{(i)}_\mu a^{(i)}_\nu\rangle = \frac{-i\,\eta_{\mu\nu}}{p^2+i0}, \quad \langle\omega^{(i)}\bar\omega^{(i)}\rangle = \frac{i}{p^2+i0}. \tag{G16}$$

Sorting field. The measure factor $\delta[\bar\nabla^2\Phi - 4\pi G(\rho-\varepsilon)]$ implies no Φ propagator and no Φ loops. The constraint is solved (with boundary data) and substituted; exterior neutrality (GAΦ) sets $\Phi\equiv0$ outside sources.

5. Background Ward/Slavnov–Taylor identities

BRST invariance $s(S_{\rm tot})=0$ yields the Zinn–Justin (Slavnov) functional identity; in the background method, the renormalized effective action $\Gamma[\bar g,\bar{\mathcal{A}}]$ is background-gauge invariant. Consequences: (i) physical S-matrix is gauge-parameter independent, (ii) standard gauge/YM Ward identities hold, (iii) no new radiative degree arises from the SSC constraint.

6. Worked 1-loop example: $U(1)_Y$ vacuum polarization and β_{g_Y}

In the background-field gauge, the renormalization of the background hypercharge field B_μ is fixed by the 1PI two-point function (vacuum polarization) $\Pi^{\mu\nu}_Y(p)$. The divergent part in dimensional regularization is transverse,

$$\Pi^{\mu\nu}_Y(p) \;\propto\; (p^\mu p^\nu - p^2\eta^{\mu\nu}),$$

and renormalizes the operator $-\frac14 B_{\mu\nu}B^{\mu\nu}$:

$$\delta\mathcal{L} = \frac14\,\delta Z_Y\,B_{\mu\nu}B^{\mu\nu}, \tag{G17}$$

$$\delta Z_Y = -\frac{g_Y^2}{16\pi^2\epsilon}\left[\frac43\sum_{\text{Weyl f}}Y^2 + \frac13\sum_{\text{complex s}}Y^2\right]. \tag{G18}$$

With three generations and one Higgs doublet,

$$\sum_{\text{Weyl f}}Y^2 = 3\times\frac{10}{3}, \qquad \sum_{\text{complex s}}Y^2 = \frac12,$$

giving

$$16\pi^2\,\beta_{g_Y} = +\frac{41}{6}\,g_Y^3,$$

in agreement with App. F. By background-gauge Ward identities, this also fixes the running of the canonically normalized background kinetic term, confirming the BRST setup.

Remark. Treating gravity as an EFT, 1-loop divergences renormalize local curvature operators (R^2, $R_{\mu\nu}R^{\mu\nu}$, etc.). Our BRST/background setup ensures the counterterm basis is unchanged by the SSC constraint (no Φ loops), and the finite parts can be fixed by matching to chosen observables in future work.

7. 1-loop power counting and counterterm basis

Treat gravity as an EFT about $\bar g$:

$$\mathcal{L}_{\rm grav,\,EFT} = \frac{2}{\kappa^2}R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + c_3R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \dots, \tag{G19}$$

$$\mathcal{L}_{\rm mix} = d_1 R\,\mathcal{L}^{(2)}_{\rm SM} + \dots \tag{G20}$$

Dimensional regularization preserves gauge/BRST identities; divergences renormalize the local operators above. SSC's constraint sector does not enlarge the counterterm basis (no Φ propagator $\Rightarrow$ no Φ-loops).

8. Quantum additions for the neutrino sector

Fields and propagators. Right-handed singlets N_R are free of gauge interactions; in momentum space $\langle N_R\,\overline{N_R}\rangle = \frac{i}{p\!\!\!/ - M_R + i0}$ (in the Dirac limit $M_R = 0$). The active neutrinos couple via the charged/neutral currents as usual through $U_{\rm PMNS}$ after diagonalization.

Vertices. From $-\,\bar{\ell}_L Y_\nu \tilde{\phi} N_R + \text{h.c.}$: a Yukawa vertex with one N_R, one lepton doublet, and one Higgs (or Goldstone in R_ξ gauge). No new gauge ghosts are introduced: N_R is a singlet.

Threshold matching (seesaw). For $\|M_R\| \gg v$, integrate out N_R at $\mu \sim \|M_R\|$:

$$\begin{aligned}\mathcal{L}_{\rm eff} &= \mathcal{L}_{\rm SM} + \tfrac{1}{2}\,(\bar{\ell}^c_L\,\tilde{\phi}^*)\,C_5(\mu)\,(\ell_L\,\tilde{\phi}) + \text{h.c.},\\ C_5(\mu) &= Y_\nu^\top\,M_R^{-1}\,Y_\nu.\end{aligned} \tag{G21}$$

Below the threshold, run $C_5(\mu)$ with the SM to low scales. Above the threshold, run Y_ν and M_R in the full renormalizable theory.

BRST and Ward identities. Adding N_R does not modify diffeo/YM BRST sectors; background Ward identities are unchanged. No anomalies are introduced by singlets.

Minimal loop checklist (neutrino sector).

1. One-loop renormalization of Y_ν (above the seesaw threshold) and C_5 (below it); confirm gauge-parameter independence.

2. Higgs and gauge two-point functions with N_R in the loop (above threshold) and with C_5 insertions (below), verifying decoupling at $\mu \ll \|M_R\|$.

3. If $M_R \neq 0$: compute $0\nu\beta\beta$ amplitude scaling with $\langle m_{\beta\beta}\rangle$ as a consistency check of Majorana limit.

9. Minimal 1-loop checklist (what to compute first)

1. **SM -functions (done):** reproduce $\beta_{g_3}, \beta_{g_2}, \beta_{g_Y}, \beta_{y_t}, \beta_\lambda$ (App. F) using background-field Feynman rules.

2. **Matter–graviton scattering (tree):** verify universal coupling $\sim \kappa h_{\mu\nu} T^{\mu\nu}$; reproduce Newtonian $1/r$ from single-graviton exchange.

3. **1-loop graviton corrections (EFT):** compute logarithms in amplitudes (e.g. matter form factors) and match to $c_{1,2,3}$.

4. **Ghost/diffeo checks:** confirm cancellation of unphysical polarizations and gauge-parameter independence of on-shell amplitudes.

10. Do-not-quantize rule for Φ

In both canonical and path-integral form, Φ is a non-propagating constraint variable. Implement it via the δ-functional in (G1) and solve it classically with boundary data; *do not* introduce a quadratic $(\nabla\Phi)^2$ term or a propagator. This preserves the classical SSC property of "no scalar radiation/dipole" at the quantum level and avoids spurious loops.

We set up the background-field/BRST renormalization framework and illustrate it with a worked one-loop example; a full renormalization analysis (all one-loop sectors and gravitational EFT counterterms) is left to future work.

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